

# Supersymmetric Unified Models

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## I. MOTIVATIONS FOR SUPERSYMMETRY

### A. Gauge Hierarchy

#### 1. Standard model

All the available experimental data at low energies ( $E < 100\text{GeV}$ ) can be adequately described by the standard model with  $SU(3) \times SU(2) \times U(1)$  gauge group. The three different gauge coupling constants originates from the three different interactions, namely, strong, weak and electromagetic interactions. The standard model has many parameters which have to be measured by experiments. There are also other conceptually unsatisfactory points as well. For instance, the electric charge is found to be quantized in nature, but this phenomena is just an accident in the standard model.

#### 2. Grand unified theories

The three interactions described by the three different gauge groups can be truly unified into a single gauge group if we choose a simple gauge group to describe all three interactions. This is realized by the grand unified theories proposed by Georgi and Glashow [1]. The grand unified theories achieved at least two good points:

- Because of simple gauge group, the electromagnetic charge is now quantized.

- Since it unifies all three couplings at high energies, it gives one constraint for three couplings. Therefore it predicts the Weinberg angle  $\theta_W$ . the prediction with a simplest possibility was found to be not very far from the experimental data. On the other hand, the unification energy  $M_G$  is now very large compared to the electroweak mass scale  $M_W$  [2]

$$\frac{M_W^2}{M_G^2} \approx \left( \frac{10^2}{10^{16}} \right)^2 \approx 10^{-28} \quad (1.1)$$

#### 3. Gravity

Even if one do not accept the grand unified theories, one is sure to accept the existence of gravitational interactions. The mass scale of the gravitational interactions is given by the Planck mass  $M_{Pl}$

$$\frac{M_W^2}{M_{Pl}^2} \approx \left( \frac{10^2}{10^{19}} \right)^2 \approx 10^{-34} \quad (1.2)$$

Now we have a problem of how to explain these extremely small ratios between the mass squared  $M_W^2$  to the fundamental mass squared  $M_G^2$  or  $M_{Pl}^2$  in eq.(1.1) or eq.(1.2). This problem is called the **gauge hierarchy problem**.

### B. Higgs Scalar

When we say **explain**, we mean that it should be given a symmetry reason. This principle is called the naturalness hypothesis [3], [4]. More precisely, the system should acquire higher symmetry as we let the small parameter going to zero. The examples of the enhanced symmetry corresponding to the small mass parameter are

$$\begin{aligned} m_{J=1/2} \rightarrow 0 &\Leftrightarrow \text{Chiral symmetry} \\ m_{J=1} \rightarrow 0 &\Leftrightarrow \text{Local gauge symmetry} \end{aligned} \quad (1.3)$$

The electroweak mass scale  $M_W$  originates from the vacuum expectation value  $v$  of the Higgs field. The scale of  $v$  in turn comes from the quadratic term of the higgs potential, namely the (negative) mass squared of the Higgs scalar  $\varphi$ . Therefore we need to give symmetry reasons for the vanishing Higgs scalar mass in order to explain the gauge hierarchy problem.

Classically the vanishing mass for scalar field does give rise to an enhanced symmetry called scale invariance. However, it is well-known that the scale invariance cannot be maintained quantum mechanically. Therefore we have only two options to explain the gauge hierarchy problem.

### 1. Technicolor model [5]

We can postulate that there is no elementary Higgs scalar at all. The Higgs scalar in the standard model has to be provided as a composite field at low energies. This option requires nonperturbative physics already at energies of the order of TeV. It has been rather difficult to construct realistic models which pass all the test at low energies especially the absence of flavor-changing neutral current. Models with composite Higgs scalar are called Technicolor models.

### 2. Supersymmetry [6]

Another option is to postulate a symmetry between Higgs scalar and a spinor field. Then we can postulate chiral symmetry for the spinor field to make it massless. The Higgs scalar also becomes massless because of the symmetry between the scalar and the spinor. This symmetry between scalar and spinor is called supersymmetry [7], [8], [9]. Contrary to the Technicolor models, we can construct supersymmetric models which can be treated perturbatively up to extremely high energies along the spirit of the grand unified theories.

Antisymmetric product of *gamma* matrices

$$\gamma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]/2 \quad (2.7)$$

Charge conjugation matrix

$$C^{-1}\gamma^\mu C = -\gamma^{\mu T}, \quad (2.8)$$

$$C^T = -C, \quad (2.9)$$

$$C^\dagger C = 1 \quad (2.10)$$

Chiral projection

$$\psi = \psi_L + \psi_R = \psi_- + \psi_+ \quad (2.11)$$

$$\psi_- = P_- \psi \equiv \frac{1 - \gamma_5}{2} \psi, \quad (2.12)$$

$$\psi_+ = P_+ \psi \equiv \frac{1 + \gamma_5}{2} \psi \quad (2.13)$$

Charge conjugate spinor

$$\psi^c \equiv C\bar{\psi}^T, \quad (2.14)$$

$$\bar{\psi} \equiv \psi\gamma_0 \quad (2.15)$$

$$(\psi^c)_\mp = (\psi_\pm^c) = C\bar{\psi}_\pm^T, \quad (2.16)$$

$$\overline{(\psi^c)_\pm} = \overline{(\psi_\mp^c)} = -\psi_\mp^T C^{-1}. \quad (2.17)$$

Majorana Spinor

$$\psi^c = \psi \rightarrow \bar{\psi} = -\psi^T C^{-1} \quad (2.18)$$

### 2. Bilinear Covariants of Majorana Spinors

$$\bar{\psi}_1 \psi_2 = \bar{\psi}_2 \psi_1, \quad (2.19)$$

$$\bar{\psi}_1 \gamma^\mu \psi_2 = -\bar{\psi}_2 \gamma^\mu \psi_1, \quad (2.20)$$

$$\bar{\psi}_1 \gamma^{\mu\nu} \psi_2 = -\bar{\psi}_2 \gamma^{\mu\nu} \psi_1, \quad (2.21)$$

$$\bar{\psi}_1 \gamma_5 \gamma^\mu \psi_2 = \bar{\psi}_2 \gamma_5 \gamma^\mu \psi_1, \quad (2.22)$$

$$\bar{\psi}_1 \gamma_5 \psi_2 = \bar{\psi}_2 \gamma_5 \psi_1 \quad (2.23)$$

$$\text{If } \psi_1 = \psi_2 \rightarrow \bar{\psi} \gamma^\mu \psi = \bar{\psi} \gamma^{\mu\nu} \psi = 0 \quad (2.24)$$

## II. INTRODUCTION TO SUPERSYMMETRY

### A. Spinors

#### 1. Convention

Metric ( $\eta_{\mu\nu}^{\text{here}} = \eta_{\mu\nu}^{\text{Wess-Bagger}} = -\eta_{\mu\nu}^{\text{Bjorken-Drell}}$ )

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.1)$$

$$P^\mu = (P^0, P^1, P^2, P^3), \quad (2.2)$$

$$P_\mu = (-P^0, P^1, P^2, P^3) \quad (2.3)$$

$$\begin{aligned} P \cdot Q &= P^\mu \eta_{\mu\nu} Q^\nu \\ &= -P^0 Q^0 + P^1 Q^1 + P^2 Q^2 + P^3 Q^3 \end{aligned} \quad (2.4)$$

$\gamma$  matrix ( $\gamma_\mu^{\text{here}} = \gamma_\mu^{\text{Wess-Bagger}} = \gamma_\mu^{\text{Bjorken-Drell}}$ )

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = -2\eta_{\mu\nu} \quad (2.5)$$

Chiral  $\gamma$  matrix

$$\begin{aligned} \gamma_5 &= \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \\ &= \gamma_5^{\text{Bjorken-Drell}} = i\gamma_5^{\text{Wess-Bagger}} \end{aligned} \quad (2.6)$$

### 3. Derivative of Grassmann Number

$$\frac{\partial}{\partial \psi_\alpha} \psi_\beta = \delta_{\alpha\beta}, \quad (2.25)$$

$$\frac{\partial}{\partial \psi_\alpha} \bar{\psi}_\beta = \delta_{\alpha\beta} \quad (2.26)$$

$$\frac{\partial}{\partial \psi_\alpha} \bar{\psi}_\beta = (C^{-1})_{\beta\alpha}, \quad (2.27)$$

$$\frac{\partial}{\partial \bar{\psi}_\alpha} \psi_\beta = (C)_{\beta\alpha} \quad (2.28)$$

$$\frac{\partial}{\partial \psi_\alpha} = \frac{\partial}{\partial \bar{\psi}_\beta} (C^{-1})_{\beta\alpha}, \quad (2.29)$$

$$\frac{\partial}{\partial \bar{\psi}_\alpha} = -(C)_{\alpha\beta} \frac{\partial}{\partial \psi_\beta} \quad (2.30)$$

$$\bar{\epsilon} \frac{\partial}{\partial \theta} = -\frac{\partial}{\partial \theta} \epsilon \quad (2.31)$$

$$\psi_+ = \begin{pmatrix} 0 \\ \eta^{*\dot{\alpha}} \end{pmatrix} \quad (2.40)$$

$$\psi^c \equiv C \bar{\psi}^T = \begin{pmatrix} \epsilon_{\alpha\beta} \eta^\beta \\ \epsilon^{\dot{\alpha}\dot{\beta}} \xi_\beta^* \end{pmatrix} = \begin{pmatrix} \eta_\alpha \\ \xi^{*\dot{\alpha}} \end{pmatrix} \quad (2.41)$$

Majorana spinor in the Weyl basis

$$\psi = \begin{pmatrix} \xi_\alpha \\ \epsilon^{\dot{\alpha}\dot{\beta}} \xi_\beta^* \end{pmatrix} = \begin{pmatrix} \xi_\alpha \\ \xi^{*\dot{\alpha}} \end{pmatrix} \quad (2.42)$$

$$\bar{\psi} = \begin{pmatrix} \epsilon^{\alpha\beta} \xi_\beta & \xi^{*\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \xi^\alpha & \xi^{*\dot{\alpha}} \end{pmatrix} \quad (2.43)$$

$$\xi^\alpha \equiv \epsilon^{\alpha\beta} \xi_\beta, \quad \eta_{\dot{\alpha}} \equiv \epsilon_{\dot{\alpha}\dot{\beta}} \eta^{\dot{\beta}} \quad (2.44)$$

### 5. Fierz Identity for Chiral Spinor

$$\theta_{\mp\alpha} \bar{\theta}_{\pm\beta} = \frac{-1}{2} \bar{\theta}_{\pm} \theta_{\mp} \left( \frac{1 \mp \gamma_5}{2} \right)_{\alpha\beta} \quad (2.45)$$

### 4. Weyl Basis and Two Component Spinor

Weyl basis of  $\gamma$  matrix

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (2.32)$$

$$\gamma_j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}, \quad (2.33)$$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.34)$$

$$C = -i\gamma_2\gamma_0 = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \quad (2.35)$$

Two component spinor notation

$$\psi = \begin{pmatrix} \xi_\alpha \\ \eta^{*\dot{\alpha}} \end{pmatrix}, \quad (2.36)$$

$$\epsilon_{\alpha\beta} \epsilon^{\beta\gamma} = \delta_\alpha^\gamma \quad (2.37)$$

$$\bar{\psi} = \begin{pmatrix} (\eta^{*\dot{\alpha}})^* & (\xi_\alpha)^* \end{pmatrix} = \begin{pmatrix} \eta^\alpha & \xi^{*\dot{\alpha}} \end{pmatrix} \quad (2.38)$$

$$\psi_- = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix} \quad (2.39)$$

## B. Supertransformation

### 1. Superfield

Distinction between bosons and fermions by  $\theta$

$\rightarrow x^\mu, \theta$  as coordinates in superspace

Superfield = field in superspace  $\rightarrow$  16 component fields

$$\begin{aligned} \Phi(x, \theta) &= C(x) + \bar{\theta}\psi(x) - \frac{1}{2}\bar{\theta}\theta N(x) - \frac{i}{2}\bar{\theta}\gamma_5\theta M(x) \\ &\quad - \frac{1}{2}\bar{\theta}\gamma^\mu\gamma_5\theta v_\mu(x) + i\bar{\theta}\theta\bar{\theta}\gamma_5\lambda(x) + \frac{1}{4}(\bar{\theta}\theta)^2 D(x) \end{aligned} \quad (2.46)$$

### 2. Supertransformation

$$\delta\theta = \epsilon, \quad \delta x^\mu = -i\bar{\epsilon}\gamma^\mu\theta \quad (2.47)$$

$$\begin{aligned} \delta\Phi(x, \theta) &= \bar{\epsilon} \left( \frac{\partial}{\partial \theta} - i\gamma^\mu\theta \frac{\partial}{\partial x^\mu} \right) \Phi(x, \theta) \\ &= - \left( \frac{\partial}{\partial \theta} - i\bar{\theta}\gamma^\mu \frac{\partial}{\partial x^\mu} \right) \epsilon \Phi(x, \theta) \\ &\equiv [\Phi(x, \theta), \bar{\epsilon}Q] = [\Phi(x, \theta), \bar{Q}\epsilon] \end{aligned} \quad (2.48)$$

### 3. Supersymmetry Algebra

$$\begin{aligned}
& [\Phi, [\bar{\epsilon}_1 Q, \bar{Q} \epsilon_2]] = [\Phi, [\bar{\epsilon}_1 Q, \bar{\epsilon}_2 Q]] \\
& = [[\Phi, \bar{\epsilon}_1 Q], \bar{\epsilon}_2 Q] - [[\Phi, \bar{\epsilon}_2 Q], \bar{\epsilon}_1 Q] \\
& = (\delta(\epsilon_2)) (\delta(\epsilon_1)) \Phi - (\delta(\epsilon_1)) (\delta(\epsilon_2)) \Phi \\
& = \left[ \left( -\frac{\partial}{\partial \theta} + i \bar{\theta} \gamma^\mu \partial_\mu \right) \epsilon_2, \bar{\epsilon}_1 \left( \frac{\partial}{\partial \theta} - i \gamma^\nu \theta \partial_\nu \right) \right] \Phi(x, \theta) \\
& = 2 \bar{\epsilon}_1 \gamma^\mu \epsilon_2 (-i \partial_\mu \Phi(x, \theta)) \\
& = 2 \bar{\epsilon}_1 \gamma^\mu \epsilon_2 [\Phi(x, \theta), P_\mu]
\end{aligned} \tag{2.49}$$

Supersymmetry algebra

$$\{Q_\alpha, \bar{Q}_\beta\} = 2(\gamma^\mu)_{\alpha\beta} P_\mu \tag{2.50}$$

$$\{Q_\alpha, Q_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} P_\mu \tag{2.51}$$

Other commutation relations

$$[Q, P_\mu] = 0, \tag{2.52}$$

$$[Q_\alpha, J^{\mu\nu}] = \frac{i}{2} (\gamma^{\mu\nu})_{\alpha\beta} Q_\beta \tag{2.53}$$

$$[P_\mu, P_\nu] = 0, \tag{2.54}$$

$$[P_\mu, J^{\nu\lambda}] = -i(\eta^{\mu\nu} P^\lambda - \eta^{\mu\lambda} P^\nu) \tag{2.55}$$

$$\begin{aligned}
[J_{\mu\rho}, J^{\nu\lambda}] &= -i(\eta^{\rho\nu} J^{\mu\lambda} + \eta^{\mu\lambda} J^{\rho\nu} \\
&\quad - \eta^{\mu\nu} J^{\rho\lambda} - \eta^{\rho\lambda} J^{\mu\nu})
\end{aligned} \tag{2.56}$$

Characteristic features of supersymmetry

1. Involving anticommutators
2. Spacetime symmetry

### C. Unitary Representation

Unitary Representation of Supersymmetry Algebra  
→ Physical Particle Content

#### 1. Massive case

1. Representation of the Poincaré group

Diagonalize  $P^\mu$

Standard frame  $P^\mu = (M, 0, 0, 0)$

Little group = Stability group of  $(M, 0, 0, 0) = SO(3)$

Angular momentum  $j$ ,  $z$  component  $m$

2. Representation of  $Q$  by combining  $(j, m)$

$$[Q, P_\mu] = 0 \tag{2.57}$$

$P^\mu$  can be diagonalized

$$[Q_\alpha, J^{\mu\nu}] = \frac{i}{2} (\gamma^{\mu\nu})_{\alpha\beta} Q_\beta \tag{2.58}$$

$Q$  changes  $j$  and  $m$  by  $\pm \frac{1}{2}$

$$\{Q_{-\alpha}, Q_{-\beta}\} = \{Q_{+\alpha}, Q_{+\beta}\} = 0 \tag{2.59}$$

$$\{Q_{-\alpha}, \overline{Q_{-\beta}}\} = \{Q_{+\alpha}, \overline{Q_{+\beta}}\} = 2M \delta_{\alpha\beta} \tag{2.60}$$

2 kinds of “fermions”

$\overline{Q_{-\alpha}}$ ,  $\alpha = 1, 2$  annihilation operator

$Q_{-\alpha}$ ,  $\alpha = 1, 2$  creation operator

Suppose  $\overline{Q_{-\alpha}}|j\rangle = 0$ ,  $\alpha = 1, 2$

$$\begin{pmatrix} & Q_{-1}|j\rangle & \\ |j\rangle & & Q_{-1}Q_{-2}|j\rangle \\ & Q_{-2}|j\rangle & \end{pmatrix} = \begin{pmatrix} & j - \frac{1}{2} & \\ j & & j \\ & j + \frac{1}{2} & \end{pmatrix} \tag{2.61}$$

(a)  $j = 0$  case  $\Rightarrow$  Chiral scalar multiplet

spin $j$	field	degree of freedom
0	two real scalar	2
1/2	a Majorana spinor	2

(b)  $j = \frac{1}{2}$  case  $\Rightarrow$  Vector multiplet

spin $j$	field	degree of freedom
0	a real scalar	1
1/2	2 Majorana spinor	4
1	a real vector	3

#### 2. Massless case

Standard frame  $P^\mu = (P, 0, 0, P)$

Little group = Stability group of  $(P, 0, 0, P)$

$$E_2 = (J^{12}, J^{01} + iJ^{23}, J^{20} + iJ^{13}) \tag{2.62}$$

Representation label by

$$J^2 = j(j+1) \quad \text{and} \quad \text{helicity } J^{12} = \pm j \tag{2.63}$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2((\gamma_0 + \gamma_3))_{\alpha\beta} P = 4P \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.64}$$

$$\{Q_{-1}, \bar{Q}_{-1}\} = 4P, \quad \bar{Q}_{-1} = Q_{-1}^* \quad (2.65)$$

Others vanishing

Multiplet

$$|\lambda\rangle \rightarrow |\lambda - \frac{1}{2}\rangle \quad (2.66)$$

CPT invariance

$$(\lambda, \lambda - \frac{1}{2}, -\lambda + \frac{1}{2}, -\lambda) \quad (2.67)$$

highest helicity	helicities of fields	name of multiplet
$\lambda = \frac{1}{2}$	$(\frac{1}{2}, 0, 0, -\frac{1}{2})$	chiral scalar multiplet
$\lambda = 1$	$(1, \frac{1}{2}, -\frac{1}{2}, -1)$	vector multiplet
$\lambda = 2$	$(2, \frac{3}{2}, -\frac{3}{2}, -2)$	graviton-gravitino multi.

#### D. Chiral Scalar Superfield

##### 1. Irreducible Representation

General superfield  $\Phi(x, \theta)$  contains too many components (8 bosons + 8 fermions) compared to the minimum number of physical degree of freedom given by the unitary representation (2 bosons + 2 fermions)

One should find a constraint consistent with supersymmetry to realize the supersymmetry in a smaller space — Key ingredient to construct field theories.

$\theta_\alpha$  : 4-components

$\theta_{\pm\alpha}$  : 2-components

If  $\Phi(x, \theta)$  is independent of  $\theta_{+\alpha}$  or  $\theta_{-\alpha}$ , the number of components is reduced to half.

$$\frac{\partial}{\partial \theta_-} \Phi(x, \theta) = 0 \quad (2.68)$$

But

$$\left\{ \frac{\partial}{\partial \theta_-}, Q \right\} \neq 0 \quad (2.69)$$

Therefore this constraint is not consistent with Supersymmetry.

Definition of Covariant derivative

$$D_\alpha \Phi(x, \theta) \equiv \left( \frac{\partial}{\partial \theta_\alpha} + i(\gamma^\mu \partial_\mu \theta)_\alpha \right) \Phi(x, \theta) \quad (2.70)$$

$$\bar{D}_\alpha \Phi(x, \theta) = \left( -\frac{\partial}{\partial \bar{\theta}_\alpha} - i(\bar{\theta} \gamma^\mu \partial_\mu)_\alpha \right) \Phi(x, \theta) \quad (2.71)$$

$$\begin{aligned} \{D_\alpha, \bar{Q}_\beta\} &= \left\{ \frac{\partial}{\partial \theta_\alpha} + i(\gamma^\mu)_{\alpha\gamma} \theta_\gamma \partial_\mu, \frac{\partial}{\partial \bar{\theta}_\beta} - i\bar{\theta}_\delta (\gamma^\mu)_{\delta\beta} \partial_\mu \right\} \\ &= i(\gamma^\mu)_{\alpha\gamma} \partial_\mu \left\{ \theta_\gamma, \frac{\partial}{\partial \bar{\theta}_\beta} \right\} - i \left\{ \frac{\partial}{\partial \theta_\alpha}, \bar{\theta}_\delta \right\} (\gamma^\mu)_{\delta\beta} \partial_\mu \\ &= i(\gamma^\mu)_{\alpha\gamma} \partial_\mu - i(\gamma^\mu)_{\alpha\gamma} \partial_\mu = 0 \end{aligned} \quad (2.72)$$

$$\{D_\alpha, Q_\beta\} = 0 \quad (2.73)$$

$$\begin{aligned} \{D_\alpha, \bar{D}_\beta\} &= \left\{ \frac{\partial}{\partial \theta_\alpha} + i(\gamma^\mu)_{\alpha\gamma} \theta_\gamma \partial_\mu, -\frac{\partial}{\partial \bar{\theta}_\beta} - i\bar{\theta}_\delta (\gamma^\mu)_{\delta\beta} \partial_\mu \right\} \\ &= -2i(\gamma^\mu)_{\alpha\beta} \partial_\mu \end{aligned} \quad (2.74)$$

$D_\alpha$  satisfies the same algebra as  $Q_\alpha$

##### 2. Chiral Projected Covariant Derivative

$$D_{\pm\alpha} = \frac{\partial}{\partial \theta_{\mp\alpha}} + i(\gamma^\mu \partial_\mu \theta_{\mp})_\alpha \quad (2.75)$$

$$\{D_{+\alpha}, D_{+\beta}\} = \{D_{-\alpha}, D_{-\beta}\} = 0 \quad (2.76)$$

$$\{D_{\pm\alpha}, D_{\mp\beta}\} = \left( \frac{1 \pm \gamma_5}{2} \gamma^\mu C \right)_{\alpha\beta} (-2i\partial_\mu) \quad (2.77)$$

Negative chiral scalar superfield

$$D_{+\alpha} \Phi(x, \theta) = 0 \quad (2.78)$$

Define

$$\begin{aligned} z^\mu &\equiv x^\mu + \frac{i}{2} \bar{\theta} \gamma^\mu \gamma_5 \theta \\ &= x^\mu - i\bar{\theta}_- \gamma^\mu \theta_- = x^\mu + i\bar{\theta}_+ \gamma^\mu \theta_+ \end{aligned} \quad (2.79)$$

Then

$$\begin{aligned} D_{+\alpha} z^\mu &= \left( \frac{\partial}{\partial \theta_{-\alpha}} + i(\gamma^\mu \partial_\mu \theta_-)_\alpha \right) (x^\mu - i\bar{\theta}_- \gamma^\mu \theta_-) \\ &= i(\gamma^\mu \theta_-)_\alpha - i(\gamma^\mu \theta_-)_\alpha = 0 \end{aligned} \quad (2.80)$$

$$D_{+\alpha} \theta_{-\beta} = 0, \quad (2.81)$$

$$D_{+\alpha} \theta_{+\beta} = - \left( \frac{1 + \gamma_5}{2} C \right)_{\alpha\beta} \neq 0 \quad (2.82)$$

Changing variables  $(x, \theta_+, \theta_-) \rightarrow (z, \theta_+, \theta_-)$

$$D_{+\alpha} \Phi(x, \theta) = 0 \implies \Phi = \Phi(z, \theta_-) \quad (2.83)$$

Namely,  $\Phi$  is independent of  $\theta_+$  if  $z$  is fixed.

Let us denote negative chiral scalar field as  $\Phi_-(z, \theta_-)$

Negative chiral scalar field can be used as a representation space of supersymmetry ( $\{Q, D\} = 0$ )

### 3. Properties of chiral scalar superfield

$$\begin{aligned}\Phi_-(z, \theta_-) &= e^{\frac{i}{2}\bar{\theta}\gamma^\mu\partial_\mu\gamma_5\theta}\Phi_-(x, \theta_-) \\ &= \left(A_-(z) + \sqrt{2\theta_+}\psi_-(z) + \bar{\theta}_+\theta_-F_-(z)\right) \\ &= e^{\frac{i}{2}\bar{\theta}\gamma^\mu\partial_\mu\gamma_5\theta}\left(A_-(x) + \sqrt{2\theta_+}\psi_-(x) + \bar{\theta}_+\theta_-F_-(x)\right)\end{aligned}\quad (2.84)$$

Chiral scalar field is complex

Degree of freedom of component fields

fields	real or complex spin	off-shell real d.o.f.	on-shell real d.o.f.
$A_-(x)$	complex scalar	2	2
$\psi_-(x)$	complex 2-comp. spinor	4	2
$F_-(x)$	complex aux. scalar	2	0

$\psi$  obeys the Dirac equation. On-shell d.o.f. is counted by  $(\text{Dirac})^2 = \text{Klein-Gordon}$ .

Product of chiral scalar superfields  $\Phi_-^1$  and  $\Phi_-^2$

$$\begin{aligned}\Phi_-^1(z, \theta_-)\Phi_-^2(z, \theta_-) &= (A_-^1(z) + \sqrt{2\theta_+}\psi_-^1(z) + \bar{\theta}_+\theta_-F_-^1(z)) \\ &\times (A_-^2(z) + \sqrt{2\theta_+}\psi_-^2(z) + \bar{\theta}_+\theta_-F_-^2(z)) \\ &= A_-^1A_-^2 + \sqrt{2\theta_+}(A_-^1\psi_-^2 + \psi_-^1A_-^2) \\ &+ \bar{\theta}_+\theta_-(F_-^1A_-^2 + A_-^1F_-^2) + 2\bar{\theta}_+\psi_-^1\bar{\theta}_+\psi_-^2 \\ &= A_-^1A_-^2 + \sqrt{2\theta_+}(A_-^1\psi_-^2 + \psi_-^1A_-^1) \\ &+ \bar{\theta}_+\theta_-(F_-^1A_-^2 + A_-^1F_-^2 - (\psi_-^1)^c\psi_-^2)\end{aligned}\quad (2.85)$$

$$\begin{aligned}(\bar{\theta}_+\psi_-^1)(\bar{\theta}_+\psi_-^2) &= ((\psi_-^1)^c\theta_-)(\bar{\theta}_+\psi_-^2) \\ &= \frac{1}{2}(\bar{\theta}_+\theta_-)(\psi_-^1C^{-1}\psi_-^2)\end{aligned}\quad (2.86)$$

Supertransformation for an “infinitesimal”  $\epsilon$

$$\begin{aligned}\delta z^\mu &= \delta x^\mu + \frac{i}{2}\delta(\bar{\theta}\gamma^\mu\gamma_5\theta) \\ &= -i\bar{\epsilon}\gamma^\mu\theta + i\bar{\epsilon}\gamma^\mu\gamma_5\theta \\ &= -2i\bar{\epsilon}_-\gamma^\mu\theta_-\end{aligned}\quad (2.87)$$

$$\begin{aligned}\delta\Phi_-(z, \theta_-) &= \left(\delta z^\mu\frac{\partial}{\partial z^\mu} + \delta\bar{\theta}_+\frac{\partial}{\partial\bar{\theta}_+}\right)\Phi_-(z, \theta_-) \\ &= \left(-2i\bar{\epsilon}_-\gamma^\mu\theta_-\frac{\partial}{\partial z^\mu} + \bar{\epsilon}_+\frac{\partial}{\partial\bar{\theta}_+}\right) \\ &\times (A_-(z) + \sqrt{2\theta_+}\psi_-(z) + \bar{\theta}_+\theta_-F_-(z)) \\ &= \sqrt{2\bar{\epsilon}_+}\psi_- + 2\bar{\epsilon}_+\theta_-F_- - 2i\bar{\epsilon}_-\gamma^\mu\theta_-\partial_\mu A_- \\ &\quad - 2\sqrt{2}\bar{\epsilon}_-\gamma^\mu\theta_-\bar{\theta}_+\partial_\mu\psi_- \\ &= \sqrt{2\bar{\epsilon}_+}\psi_- + \sqrt{2\theta_+}\sqrt{2}(\epsilon_-F_- + i\gamma^\mu\epsilon_-\partial_\mu A_-) \\ &\quad + \bar{\theta}_+\theta_-\sqrt{2\bar{\epsilon}_-}i\gamma^\mu\partial_\mu\psi_-\end{aligned}\quad (2.88)$$

Therefore

$$\begin{aligned}\delta A_- &= \sqrt{2\bar{\epsilon}_+}\psi_- \\ \delta\psi_- &= \sqrt{2}(\epsilon_-F_- + i\gamma^\mu\epsilon_-\partial_\mu A_-) \\ \delta F_- &= i\sqrt{2\bar{\epsilon}_-}\gamma^\mu\partial_\mu\psi_-\end{aligned}\quad (2.89)$$

### 4. Positive Chiral Scalar Field

$$D_-\Phi_+ = 0, \quad (2.90)$$

$$z^{*\mu} = x^\mu - \frac{i}{2}\bar{\theta}\gamma^\mu\gamma_5\theta \quad (2.91)$$

$$\begin{aligned}\Phi_+ &= \Phi_+(z^*, \theta_+) \\ &= A_+(z^*) + \sqrt{2\theta_-}\psi_+(z^*) + \bar{\theta}_-\theta_+F_+(z^*)\end{aligned}\quad (2.92)$$

Product of positive chiral and negative chiral scalar fields is a general superfield (without a definite chirality)

Complex conjugation changes the chirality

$$\begin{aligned}(\Phi_-(z, \theta_-))^* &= e^{-\frac{i}{2}\bar{\theta}\gamma^\mu\partial_\mu\gamma_5\theta} \\ &\times (A_-^*(x) + \sqrt{2\theta_-}(\psi_-)^c(x) + \bar{\theta}_+\theta_-F_-^*(x))\end{aligned}\quad (2.93)$$

## E. Supersymmetric Field Theory

### 1. Lagrangian with Chiral Scalar Fields

Lagrangian invariant under supersymmetry transformation up to a total divergence:

1. Two possibilities

(a)  $D$ -term of general superfield  $\Phi$

$$[\Phi]_D = \frac{1}{8}(\bar{D}D)^2\Phi \quad (2.94)$$

(b)  $F_\pm$ -term of chiral scalar superfield  $\Phi_\pm$

$$[\Phi_\pm]_F = -\frac{1}{4}\bar{D}D\Phi_\pm \quad (2.95)$$

2. Dimensional analysis

$$[\Phi_\pm] = M^1 \quad (2.96)$$

$$[\theta_\alpha] = L^{\frac{1}{2}} = M^{-\frac{1}{2}}, \quad (2.97)$$

$$[D_\alpha] = [\bar{D}_\alpha] = M^{\frac{1}{2}} \quad (2.98)$$

3. Renormalizable Lagrangian (in 4-dimension)  
operators with dimension  $\leq 4$ .

(a) D-type:

$$(\overline{D}D)^2 \cdot \Phi_{1+}\Phi_{2-} \quad (2.99)$$

Dimension  $[\overline{D}D] = M^2$

(b) F-type:

$$(\overline{D}D)(a\Phi_- + b\Phi_{1-}\Phi_{2-} + c\Phi_{1-}\Phi_{2-}\Phi_{3-}) \quad (2.100)$$

Since  $\overline{D}D$  has dimension  $M^1$ , up to third order polynomials of chiral scalar superfields of one chirality are renormalizable.

4. General Lagrangian with a single chiral scalar field

$$L = L_{\text{kin}} + L_{\text{int.}} \quad (2.101)$$

$$\begin{aligned} L_{\text{kin}} &= \frac{1}{32}(\overline{D}D)^2 \Phi_-^* \Phi_- \\ &= \frac{1}{4}\partial^2 A_-^* A_- - \frac{1}{2}\partial_\nu A_-^* \partial^\nu A_- + \frac{1}{4}A_-^* \partial^2 A_- \\ &\quad + F_-^* F_- + \frac{1}{2}\overline{\psi_-} i\gamma^\mu \partial_\mu \psi_- - \frac{1}{2}\partial_\mu \overline{\psi_-} i\gamma^\mu \psi_- \\ &= -\partial_\nu A_-^* \partial^\nu A_- + \overline{\psi_-} i\gamma^\mu \partial_\mu \psi_- + F_-^* F_- \\ &\quad + \text{total derivatives} \end{aligned} \quad (2.102)$$

$$\begin{aligned} L_{\text{int.}} &= -\frac{1}{4}\overline{D}D \left( \frac{\sqrt{2}}{3}f\Phi_-^3 + \frac{m}{2}\Phi_-^2 + \text{h.c.} \right) \\ &= \sqrt{2}f \left( F_- A_-^2 - (\overline{\psi_-})^c \psi_- A_- \right) \\ &\quad + m \left( F_- A_- - \frac{1}{2}(\overline{\psi_-})^c \psi_- \right) + \text{h.c.} \end{aligned} \quad (2.103)$$

Elimination of auxiliary fields  $F$  from  $L$

Euler eq. for  $F_-$

$$F_-^* + \sqrt{2}fA_-^2 + mA_- = 0 \quad (2.104)$$

$$\begin{aligned} L &\rightarrow -\partial_\nu A_-^* \partial^\nu A_- + \frac{1}{2}\overline{\psi_-} i\gamma^\mu \partial_\mu \psi_- - \frac{m}{2}\overline{\psi_-} \psi_- \\ &\quad - \left( \sqrt{2}f(\overline{\psi_-})^c \psi_- A_- + \text{h.c.} \right) \\ &\quad - |\sqrt{2}fA_-^2 + mA_-|^2 \end{aligned} \quad (2.105)$$

$m$  : mass of a Majorana spinor  $\psi$  and a complex scalar  $A$

$f$  : Yukawa coupling and  $|A_-^2|^2$  coupling

5. Feynman diagram calculation is facilitated by superfield perturbation

$$-\frac{1}{4}\overline{D}D \approx \frac{1}{2}d\theta_1 d\theta_2 \equiv d^2\theta \quad (2.106)$$

$$\frac{1}{32}(\overline{D}D)^4 \approx \frac{1}{4}d\theta_1 d\theta_2 d\theta_3 d\theta_4 \equiv d^4\theta \quad (2.107)$$

## 2. Supersymmetric Gauge Theory

1. Gauge Transformation

Ordinary local gauge transformation

$$\psi(x) \rightarrow e^{-i\Lambda^a(x)T^a} \psi(x) \quad (2.108)$$

Supersymmetric extension

$$\begin{array}{ccc} \text{matter} & & \text{chiral scalar superfield} \\ \psi(x) & \rightarrow & \Phi_-(x, \theta) \end{array} \quad (2.109)$$

$x$ -dependent gauge function  $\Lambda(x)$  is generalized to a chiral scalar superfield  $\Lambda_-(x, \theta)$

$$\Lambda(x) \rightarrow \Lambda_-(x, \theta) \quad (2.110)$$

Supersymmetrized local gauge transformation

$$\Phi_-(x, \theta) \rightarrow \exp(-i\Lambda_-(x, \theta)T^a) \Phi_-(x, \theta) \quad (2.111)$$

using gauge function superfield with the same chirality

$$\Phi_-^\dagger(x, \theta) \rightarrow \Phi_-^\dagger(x, \theta) \exp(i\Lambda_-(x, \theta)^* T^a) \quad (2.112)$$

2. Gauge Invariant Kinetic Term for Matter Fields

(a) A General Superfield for Gauge Boson and Gaugino

$$e^{2gV^a T^a} \quad (2.113)$$

Gauge transformation

$$e^{2gV^a T^a} \rightarrow e^{-i\Lambda_-(x, \theta)^* T^a} e^{2gV^a T^a} e^{i\Lambda_-(x, \theta) T^a} \quad (2.114)$$

(b) Kinetic Term for a Chiral Scalar Field  $\Phi_-$

$$L_{\text{kin.}} = \frac{1}{32}(\overline{D}D)^2 (\Phi_-^\dagger e^{2gV^a T^a} \Phi_-) \quad (2.115)$$

is gauge invariant

The general superfield  $V^a$  is dimensionless and real

$$V^{a*} = V^a \quad (2.116)$$

## 3. Gauge Transformation

1. Gauge transformation in components

U(1) case

$$V \rightarrow V + \frac{i}{2g}(\Lambda_- - \Lambda_-^*) \quad (2.117)$$

In terms of components

$$\begin{aligned}
V(x, \theta) \equiv & C(x) + i\bar{\theta}_+\chi_-(x) - i\bar{\theta}_-\chi_+(x) \\
& + \frac{i}{2}\bar{\theta}_+\theta_-(M + iN) - \frac{i}{2}\bar{\theta}_-\theta_+(M - iN) \\
& - \bar{\theta}_+\gamma^\mu\theta_+v_\mu(x) \\
& + i\bar{\theta}_+\theta_-\bar{\theta}_-(\lambda_+ + \frac{i}{2}\gamma^\mu\partial_\mu\chi_-) \\
& - i\bar{\theta}_-\theta_+\bar{\theta}_+(\lambda_- + \frac{i}{2}\gamma^\mu\partial_\mu\chi_+) \\
& + \frac{1}{2}\bar{\theta}_+\theta_-\bar{\theta}_-\theta_+\left(D + \frac{1}{2}\partial^2C\right) \quad (2.118)
\end{aligned}$$

$$\begin{aligned}
C &\rightarrow C + \frac{i}{2g}(A_- - A_-^*) \\
\chi_- &\rightarrow \chi_- + \sqrt{2}\frac{1}{2g}\psi_-, \\
\chi_+ &\rightarrow \chi_+ + \sqrt{2}\frac{1}{2g}(\psi_-)^c \\
M &\rightarrow M + \frac{1}{2g}(F_- + F_-^*), \\
N &\rightarrow N + \frac{i}{2g}(F_- - F_-^*) \quad (2.119)
\end{aligned}$$

$C, \chi, M, N$  can be gauged away

$$v^\mu \rightarrow v^\mu + \frac{1}{2g}\partial^\mu(A_- + A_-^*) \quad (2.120)$$

$v^\mu$  is an ordinary gauge field

$$\lambda \rightarrow \lambda \quad D \rightarrow D \quad (2.121)$$

$\lambda, D$  are gauge invariant.

## 2. Wess-Zumino gauge

Eliminate  $C, \chi, M, N$  by choosing  $\Lambda_-$

$$\begin{aligned}
V_{WZ} = & -\bar{\theta}_+\gamma^\mu\theta_+v_\mu(x) + i\bar{\theta}_+\theta_-\bar{\theta}_-\lambda_+ \\
& -i\bar{\theta}_-\theta_+\bar{\theta}_+\lambda_- + \frac{1}{2}\bar{\theta}_+\theta_-\bar{\theta}_-\theta_+D(x) \quad (2.122) \\
= & -\frac{1}{2}\bar{\theta}_+\gamma^\mu\gamma_5\theta_+v_\mu(x) + i\bar{\theta}_+\theta_-\bar{\theta}_-\lambda_+ + \frac{1}{4}(\bar{\theta}_+\theta_-)^2D(x)
\end{aligned}$$

Wess-Zumino gauge is not manifestly supersymmetric.

However, particle content is most easily seen.

### 4. Supersymmetric Gauge Field Strength

Gauge field strength = gauge covariant building block

$\lambda^a(x)$  is the gauge covariant field with lowest dimension

Derivative  $\rightarrow D_{\pm\alpha}$

$$W_{--\alpha} \equiv \frac{-1}{8g}(\bar{D}_-D_+)\left(e^{-2gV^aT^a}D_{-\alpha}e^{2gV^aT^a}\right) \quad (2.123)$$

The first  $-$  suffix denotes negative chiral projection for the index  $\alpha$ .

The second  $-$  suffix denotes negative chiral superfield.

$$D_+W_{--\alpha} = 0 \quad (2.124)$$

$W_{--\alpha}$  is gauge covariant

$$W_{--\alpha} \rightarrow e^{-i\Lambda_-^aT^a}W_{--\alpha}e^{i\Lambda_-^aT^a} \quad (2.125)$$

Similarly positive chiral field strength is given by

$$W_{++\alpha} = \frac{-1}{8g}(\bar{D}_+D_-)(e^{2gV^aT^a}D_{+\alpha}e^{-2gV^aT^a}) \quad (2.126)$$

Kinetic term for vector superfield is given by

$$L_{\text{gauge}} = \frac{-1}{16}\bar{D}_+D_-\left(\overline{W_{++}}^aW_{--}^a\right) + \text{h.c.} \quad (2.127)$$

In the Wess-Zumino gauge

$$W_{--} = e^{\frac{i}{2}\bar{\theta}_+\gamma^\mu\partial_\mu\gamma_5\theta_-} \quad (2.128)$$

$$\times \left[ i\lambda_- - \left( D + \frac{i}{2}\gamma^{\mu\nu}v_{\mu\nu} \right) \theta_- + \bar{\theta}_+\theta_-(\gamma^\mu\nabla_\mu\lambda_-) \right]$$

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + ig[v_\mu, v_\nu] \quad (2.129)$$

$$\nabla_\mu\lambda_- = \partial_\mu\lambda_- + ig[v_\mu, \lambda_-] \quad (2.130)$$

$$\begin{aligned}
L_{\text{gauge}} = & \frac{1}{2}\overline{\lambda_+}^a(i\gamma^\mu\nabla_\mu\lambda_+)^a - \frac{1}{8}v_{\mu\nu}^av^{a\mu\nu} + \frac{1}{4}D^aD^a \\
& + \frac{i}{16}\epsilon^{\mu\nu\rho\tau}v_{\mu\nu}^av_{\rho\tau}^a + \text{h.c.} \quad (2.131) \\
= & \frac{1}{2}\overline{\lambda}^a(i\gamma^\mu\nabla_\mu\lambda)^a - \frac{1}{4}v_{\mu\nu}^av^{a\mu\nu} + \frac{1}{2}D^aD^a
\end{aligned}$$

$D^a$  is an auxiliary field

### 5. Gauge Interaction in the Wess-Zumino Gauge

$$\begin{aligned}
L_{\text{kin.of}\Phi_-} = & -(\nabla_\mu A_-)^\dagger\nabla^\mu A_- + \overline{\psi_-}i\gamma^\mu\nabla_\mu\psi_- \\
& + F_-^\dagger F_- + i\sqrt{2}g(A_-^\dagger T^a\overline{\psi_+}\lambda_-^a - \overline{\lambda_-}^a\psi_+T^aA_-) \\
& + gA_-^\dagger D^aT^aA_- \quad (2.132)
\end{aligned}$$

$$\nabla_\mu A_- = \partial_\mu A_- + igv_\mu^aT^aA_- \quad (2.133)$$



Eliminating  $D$  by Euler eq. from  $L_{gauge} + L_{kin}$

$$D^a + gA_-^\dagger T^a A_- = 0 \quad (2.134)$$

$$\begin{aligned} \frac{1}{2}D^a D^a + gA_-^\dagger D^a T^a A_- \\ = -\frac{1}{2} \sum_a g^2 |A_-^\dagger T^a A_-|^2 \end{aligned} \quad (2.135)$$

This is the  $D$ -term of the scalar potential

$U(1)$   $\xi$ -term (Fayet-Iliopoulos term)

$$L_\xi = \frac{1}{16}(\bar{D}D)^2 \xi V = \xi D \quad (2.136)$$

$$[\xi] = M^2 \quad (2.137)$$

### III. SUPERSYMMETRIC $SU(3) \times SU(2) \times U(1)$ MODEL

#### A. Yukawa Coupling

##### 1. Nonsupersymmetric Standard Model

In the nonsupersymmetric  $SU(2) \times U(1)$  model, we have left-handed quark doublet  $q_j$ , the right-handed  $u$ -type quark  $u_{Ri}$  and  $d$ -type quark  $d_{Ri}$ , left-handed lepton doublet  $l_j$ , the right-handed electron  $e_{Ri}$ , together with Higgs doublets. We shall denote the generation index by lower suffixes  $i, j, \dots$ . We also denote the Higgs doublets to give the masses to the  $u$ -type ( $d$ -type) quark as  $\varphi_u$  ( $\varphi_d$ ). We can write down the Yukawa interaction between quarks, leptons and Higgs fields in terms of the Yukawa couplings  $f$  as

$$\begin{aligned} L_{Yukawa} \\ = f_u^{ij} \overline{u_{Ri}} \varphi_u^T \varepsilon q_j + f_d^{ij} \overline{d_{Ri}} \varphi_d^T \varepsilon q_j + f_e^{ij} \overline{e_{Ri}} \varphi_d^T \varepsilon l_j \end{aligned} \quad (3.1)$$

where

$$q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}, \quad l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}, \quad (3.2)$$

$$\varphi_u = \begin{pmatrix} \varphi_u^+ \\ \varphi_u^0 \end{pmatrix}, \quad \varphi_d = \begin{pmatrix} \varphi_d^0 \\ \varphi_d^- \end{pmatrix} \quad (3.3)$$

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (3.4)$$

In the nonsupersymmetric model, we can choose the Higgs doublet  $\varphi_u$  and  $\varphi_d$  to be the complex conjugate of each other

$$\varphi_u = \varepsilon \cdot \varphi_d^* \quad (3.5)$$

This is the choice in the minimal standard model.

##### 2. Supersymmetric Standard Model

In the supersymmetric models, the Yukawa interaction has to come from the  $F$ -type interaction. This implies that the superfield participating in the Yukawa interaction should have the same chirality. Therefore the choice (3.5) cannot be taken, since the chirality changes by complex conjugation. Namely the Higgs superfield  $H_u$  corresponding to  $\varphi_u$  and  $H_d$  corresponding to  $\varphi_d$  have to be different.

$$H_u \neq \varepsilon \cdot H_d^* \quad (3.6)$$

The supersymmetric Yukawa interaction is given by

$$L_{Yukawa} = -\frac{1}{4} \bar{D}DW(\Phi) + h.c. \quad (3.7)$$

$$\begin{aligned} W \\ = f_u^{ij} U_i^c H_u^T \varepsilon Q_j + f_d^{ij} D_i^c H_d^T \varepsilon Q_j + f_e^{ij} E_i^c H_d^T \varepsilon L_j \end{aligned} \quad (3.8)$$

where we denoted the negative chiral scalar superfield by capital letters and the charge conjugate of the positive chiral scalar superfield in terms of the upper suffix  $c$ .

#### B. Particle Content

Now we find that we need at least a pair of Higgs doublet superfield, we will list the minimal particle content of the supersymmetric standard model. We shall use the convention for the  $U(1)$  charge  $Y$  as

$$Q = I_3 + Y \quad (3.9)$$

Let us note that the Higgsino (chiral fermions associated with the Higgs scalar) in general introduces the anomaly in gauge currents. The simplest way out of such anomaly problem is to introduce the Higgsino doublet in pairs. Then the anomaly coming from  $\tilde{\varphi}_u$  and  $\tilde{\varphi}_d$  always cancel each other. This is another reason to introduce pair of Higgs doublet superfield  $H_u$  and  $H_d$ .

	$J = 1$	$J = 1/2$	$J = 0$	I	Y	$SU(3)$
Gauge fields						
$G$	$g_\mu$	$\tilde{g}$				
$W$	$W_\mu$	$\tilde{W}$				
$B$	$B_\mu$	$\tilde{B}$				
Higgs field						
$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$		$\tilde{\varphi}_u$	$\varphi_u$	$\frac{1}{2}$	$\frac{1}{2}$	
$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$		$\tilde{\varphi}_d$	$\varphi_d$	$\frac{1}{2}$	$-\frac{1}{2}$	
Quark field						
$Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}$		$q_i$	$\tilde{q}_i$	$\frac{1}{2}$	$\frac{1}{6}$	3
$U_i^c$		$u_i^c$	$\tilde{u}_i^c$	0	$-\frac{2}{3}$	$3^*$
$D_i^c$		$d_i^c$	$\tilde{d}_i^c$	0	$\frac{1}{3}$	$3^*$
Lepton field						
$L_i = \begin{pmatrix} N_i \\ E_i \end{pmatrix}$		$l_i$	$\tilde{l}_i$	$\frac{1}{2}$	$-\frac{1}{2}$	
$E_i^c$		$e_i^c$	$\tilde{e}_i^c$	0	1	
$(N_i^c)$		$\nu_i^c$	$\tilde{\nu}_i^c$	0	0	)

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